X-602-76-35

PREPRINT

NASA TM X- 71072

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(NASA-TM-X-71072) STRONG SCINTILLATIONS IN ASTROPHYSICS. 4. CROSS-CORRELATION BETWEEN DIFFERENT FREQUENCIES AND FINITE BANDWIDTH EFFECTS (NASA) 29 p HC $4.00 CSCL 03B

N76-19024

Unclas

G3/90 20038

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FEBRUARY 1976

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

STRONG SCINTILLATIONS IN ASTROPHYSICS

IV. CROSS-CORRELATION BETWEEN DIFFERENT FREQUENCIES AND FINITE BANDWIDTH EFFECTS

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ABSTRACT

We calculate the cross-correlation of the intensity fluctuations between different frequencies and finite bandwidth effects on the intensity correlations based on the Markov approximation discussed in Paper I. Our results may be applied to quite general turbulence spectra for an extended turbulent medium. Calculations of the cross-correlation function and of finite bandwidth effects are explicitly carried out for both Gaussian and Kolmogorov turbulence spectra. The increases of the correlation scale of intensity fluctuations are different for these two spectra and the difference can be used to determine whether the interstellar turbulent medium has a Gaussian or a Kolmogorov spectrum.

I. INTRODUCTION

Frequency correlations of pulsar scintillations caused by the irregularities of interstellar medium have been observed (Scott and Collins 1968, Komesaroff et al. 1971, Salpeter 1969, Rickett 1969, Lang 1971, and Sutton 1971) and the effect of finite receiver bandwidth on the observed scintillations has also been detected (Rickett 1969). A theoretical interpretation of these phenomena requires a knowledge of the cross-correlation function of intensity fluctuations between different transverse coordinates and different frequencies, $P_I(z, \underset{\sim}{\zeta}_1, k_1, \underset{\sim}{\zeta}_2, k_2)$. (Lee 1974).

Little (1968) and Lovelace (1970) calculated the effects of finite bandwidth on the observed intensity scintillations based on a "thin screen" model for a Gaussian irregularity spectrum. The "thin screen" approximation is not necessarily realistic in interstellar scintillations since the interstellar irregularities may not be confined to a small region between source and observer. Shishov (1975) calculated the frequency correlations for both the "thin screen" model and an extended medium. However, his results do not include finite bandwidth effects on the correlation function of intensity fluctuations and are limited to a turbulent medium with a single-scale power spectrum. Budden and Uscinski (1970, 1971) also calculated the effects of finite bandwidth on the scintillations of extended radio sources and found them to be practically negligible, in contrast to the conclusion of Little (1968).

In Paper I of this series (Lee and Jokipii 1975a), we developed a theory of strong scintillations, which was applicable to a number of pulsar signals. Angular broadening and phase correlation were discussed in this first paper. In Paper II (Lee and Jokipii 1975b) we applied this theory to the temporal broadening of pulses. In Paper III (Lee and Jokipii 1975c), a theory of intensity fluctuations for monochromatic waves was presented. In the present paper, we apply the techniques of Paper I to calculate the cross-correlations of intensity fluctuations between different wave-numbers and different transverse coordinates $P_I\ (z,\ \underset{\sim}{\zeta}_1,\ k_1,\ \underset{\sim}{\zeta}_2,\ k_2)$, and finite bandwidth effects on the intensity correlation functions for an extended medium. The calculations are carried out for both Gaussian and power-law (Kolmogorov) spectra of the turbulent medium. We also point out the differences in finite bandwidth effects between a Gaussian spectrum and a power-law spectrum. This difference can be used to determine the nature of the interstellar irregularity spectrum.

II. GENERAL CONSIDERATIONS

We consider the propagation for a wave E $(\underset{\sim}{r}, t)$ whose Fourier component

$$E_\omega(\underset{\sim}{r}, t) = \Phi_\omega(\underset{\sim}{r}) \quad e^{-i\omega t} \tag{1}$$

obeys the scalar wave equation

$$\nabla^2 \Phi_\omega(\underset{\sim}{r}) + \frac{\omega^2}{c^2} \varepsilon_\omega(\underset{\sim}{r}) \Phi_\omega(\underset{\sim}{r}) = 0 \tag{2}$$

Here $(\omega/2\pi)$ is the frequency of the Fourier component $\Phi_\omega(\underset{\sim}{r})$ c is the speed of light, and $\varepsilon_\omega(\underset{\sim}{r})$ is the refractive index in which the wave propagates. The refractive index $\varepsilon_\omega(\underset{\sim}{r})$ is a random function and depends on both the position $\underset{\sim}{r}$ and the wave frequency ω. As an example, we consider in this paper the propagation of the high frequency waves with $\omega \gg \omega_p$, the plasma frequency of the medium, in the plasma medium. This applies to the propagation of the radio waves in the ionosphere, in interplanetary space, or in the interstellar medium. If N_e is the electron density, then we have

$$\varepsilon_\omega(\underset{\sim}{r}) = 1-(\omega_p^2/\omega^2) \tag{3}$$

and

$$\omega_p^2 = 4\pi N_e e^2/m, \tag{4}$$

where e is the charge and m is the mass of an electron (Paper I).

Now let N_e and $\varepsilon_\omega(\underset{\sim}{r})$ vary randomly, and let $< \quad >$ denote an average over an ensemble of propagation volumes. Following Lee (1974), we define

$$N_e(\underset{\sim}{r}) = \langle N_e(\underset{\sim}{r})\rangle + \delta N_e(\underset{\sim}{r})$$

$$\varepsilon_\omega(\underset{\sim}{r}) = \langle \varepsilon_\omega(\underset{\sim}{r})\rangle + \delta\varepsilon_\omega(\underset{\sim}{r})$$

$$\beta(\underset{\sim}{r}) = -4\pi e^2 \delta N_e(\underset{\sim}{r})/mc^2$$

$$\varepsilon_k(\underset{\sim}{r}) = \delta\varepsilon_\omega/\langle\varepsilon_\omega\rangle = \beta(\underset{\sim}{r})/k^2$$

$$k = \frac{\omega}{c}\langle\varepsilon_\omega\rangle^{1/2} = \frac{\omega}{c}\left(1 - \frac{4\pi\langle N_\omega\rangle e^2}{m\omega^2}\right)^{\frac{1}{2}}$$

$$\underset{\sim}{r} = (z, \underset{\sim}{\zeta})$$

$$\underset{\sim}{\zeta} = (x, y)$$

and

$$S = (\underset{\sim}{\zeta}, k),$$

where k is the wave-number and $\underset{\sim}{\zeta}$ is the transverse position.

We also define

$$\Phi_\omega(\underset{\sim}{r}) = u(z, \underset{\sim}{\zeta}, k)\, e^{ikz} \qquad (5)$$

for a plane wave propagating in the z direction. Then we have from Eq. (2)

$$2ik\frac{\partial u}{\partial z}(z, \underset{\sim}{\zeta}, k) + \left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) u(z,\underset{\sim}{\zeta}, k) + \beta(\underset{\sim}{r})\, u(z,\underset{\sim}{\zeta},k) = 0, \qquad (6)$$

where now $\beta(\underset{\sim}{r})$ is a wave-frequency independent random variable with zero mean.

As in Papers I, II, and III, we will consider here a situation in which each Fourier component $E_\omega(\underset{\sim}{r})$ be a plane wave propagates freely from the source along the +z direction until it strikes the turbulent medium at the plane z=0. The observers are situated at the plane $z > 0$.

The intensity of the Fourier component with wave-number k and observed at position $(z, \underset{\sim}{\zeta})$ is $I(z,\underset{\sim}{\zeta}, k) = |u(z, \underset{\sim}{\zeta}, k)|^2$. The normalized cross-correlation function of intensity fluctuation between two Fourier components observed respectively at $(z, \underset{\sim}{\zeta}_1)$ and $(z, \underset{\sim}{\zeta}_2)$

can be written as

$$P_I(z, \underset{\sim}{\zeta}_1, k_1, \underset{\sim}{\zeta}_2) = \frac{\langle I_1 I_2\rangle}{\langle I_1\rangle\langle I_2\rangle} - 1$$

$$= \frac{\langle u_1 u_1^* u_2 u_2^*\rangle}{\langle u_1 u_1^*\rangle\langle u_2 u_2^*\rangle} - 1, \tag{7}$$

where $I_i = I(z, \underset{\sim}{\zeta}_i, k_i)$ and $u_i = u(z, \underset{\sim}{\zeta}_i, k_i)$ for $i = 1,2$. In particular, if we set $k_1 = k_2$, P_I becomes the spatial intensity correlation function, which was discussed in Paper III. If we set $\underset{\sim}{\zeta}_1 = \underset{\sim}{\zeta}_2$, P_I is the frequency correlation function of intensity fluctuations.

The cross-correlation function P_I in Eq. (7) is for detectors with zero bandwidth. We consider now the case of detectors with a finite bandwidth. Let $f_K(k)$ be the normalized intensity response of the bandpass, i.e.

$$\int f_K(k)\, dk = 1$$

Then we have the observed intensity at $(z, \underset{\sim}{\zeta})$

$$I(z, \underset{\sim}{\zeta}) = \int |u(z, \underset{\sim}{\zeta}, k)|^2 f_K(k)\, dk.$$

The correlation function of intensity fluctuation measured with a bandwidth function $f_K(k)$ is

$$Q_I(z, \underset{\sim}{\zeta}_1, \underset{\sim}{\zeta}_2, K) = \frac{\iint \langle u_1 u_1^* u_2 u_2^*\rangle f_K(k_1) f_K(k_2)\, dk_1 dk_2}{\left[\int \langle u_1 u_1^*\rangle\ f_K(k_1)\, dk_1\right]^2} - 1 \tag{8}$$

where the parameter K in $Q_I(z, \zeta_1, \zeta_2, K)$ denotes the characteristics of the bandwidth function f_K to be specified in Section IV. The scintillation index $m_z(K)$ as a function of K is defined as

$$m_z^2(K) = Q_I(z, \underset{\sim}{\zeta}_1, \underset{\sim}{\zeta}_2 = \underset{\sim}{\zeta}_1, K). \tag{9}$$

It is the purpose of this paper to calculate P_I, Q_I and $m_z(K)$. From Eqs. (7), (8) and (9), it is clear that in order to obtain these quantities, one has to calculate the fourth order correlation function $\Gamma_{2,2}$ (z, S_1, S_2 S_3, S_4) = $\langle u_1\ u_2\ u_3{}^*\ u_4{}^*\rangle$. A transport equation for $\Gamma_{2,2}$ with different wave-numbers has been given in Eq.(42) of Lee (1974). This transport equation is very complicated and will not be solved here.

However, as was shown in Paper III, the fourth moment $\Gamma_{2,2}$ of the complex amplitude u with same wave numbers (monochromatic waves) is simply related to the second moments in the strong scintillation region. It can also be shown that this relationship between the fourth and second moments of the field u is the same as for the case in which the complex amplitude u has a joint-normal distribution. Hence it is reasonable to assume that in the strong scintillation, the fourth moment $\Gamma_{2,2}$ of the complex field u with different wave-numbers is also related to the first and second moments as if the complex u has a joint-normal distribution. Under this assumption, Eqs. (7) and (8) can be written respectively as

$$P_I\ (z,\ \underset{\sim}{\zeta}_1,\ k_1,\ \underset{\sim}{\zeta}_2,\ k_2) = \frac{\langle u(z,\ \underset{\sim}{\zeta}_1,\ k_1)\ u^*(z,\ \underset{\sim}{\zeta}_2,\ k_2)\rangle^2}{\langle u_1 u_1{}^*\rangle\ \langle u_2\ u_2{}^*\rangle} \qquad (7')$$

and

$$Q_I(z,\ \underset{\sim}{\zeta}_1,\ \underset{\sim}{\zeta}_2,\ K) = \frac{\iint |\langle u_1\ u_2{}^*\rangle|^2 f_K(k_1) f_K(k_2) dk_1 dk_2}{[\int \langle u_1 u_1{}^*\rangle f_K(k_1) dk_1]^2} \qquad (8')$$

by noting that $\langle u\rangle$ and $\langle u_1 u_2\rangle$ are negligibly small in the region of strong scintillations. Thus instead of solving the transport equation for the fourth moment $\Gamma_{2,2}$, we will determine $\Gamma_{1,1}$ (z, $\underset{\sim}{\zeta}_1$, k_1, $\underset{\sim}{\zeta}_2$, k_2) = $\langle u_1\ u_2{}^*\rangle$, which is the correlation between u_1 and $u_2{}^*$ at different transverse coordinates and with different wave-numbers.

III. CROSS-CORRELATION FUNCTION OF INTENSITY FLUCTUATIONS

A transport equation for $\Gamma_{1,1}(z, \underset{\sim}{\zeta}_1, k_1, \underset{\sim}{\zeta}_2, k_2) = \langle u_1\, u_2^* \rangle$ can be obtained from Eq. (6) under the "Quasi-optics" and "Markov random process" approximations (Lee, 1974), and one has

$$\frac{\partial \Gamma_{1,1}}{\partial z} = \frac{i}{2}\left(\frac{\nabla_1^2}{k_1} - \frac{\nabla_2^2}{k_2}\right)\Gamma_{1,1} - \frac{1}{4}\left\{\left(\frac{1}{k_1^2} + \frac{1}{k_2^2}\right)A_\beta(0) - \frac{2A_\beta(\underset{\sim}{\zeta}_1 - \underset{\sim}{\zeta}_2)}{k_1\, k_2}\right\}\Gamma_{1,1} \tag{10}$$

where $\nabla_i^2 = \nabla_{\underset{\sim}{\zeta}_i}^2$, $i = 1,2$, and

$$A_\beta\,(\underset{\sim}{\zeta}_1 - \underset{\sim}{\zeta}_2) = \frac{1}{2}\int_{-\infty}^{\infty} \langle \beta\,(z, \underset{\sim}{\zeta}_1)\,\beta(z', \underset{\sim}{\zeta}_2)\rangle\, dz' \tag{11}$$

The solution of Eq. (9) was discussed in Paper II in relation to calculations of the temporal broadening of pulses. Following Paper II, we define $k_1 = k + \frac{\Delta k}{2}$, $k_2 = k - \frac{\Delta k}{2}$, and assume Δk is small and $|\Delta k| \ll k$. We also normalize the intensity of incoming wave such that $u(z = 0, \underset{\sim}{\zeta}, k) = 1$. Then for a statistically homogeneous medium, one can write (Paper II)

$$\Gamma_{1,1}\,(z, \underset{\sim}{\zeta}_1, k_1, \underset{\sim}{\zeta}_2, k_2) = \Gamma_D\,(z, \underset{\sim}{\zeta}, \Delta k)\,\Gamma_R\,(z, \Delta k) \tag{12}$$

where $\underset{\sim}{\zeta} = \underset{\sim}{\zeta}_1 - \underset{\sim}{\zeta}_2$

$$\Gamma_R\,(z, \Delta k) = \exp\left\{-\frac{\Delta k^2}{4k^2}\,A_\beta(0)\,z\right\} \tag{12a}$$

and $\Gamma_D\,(z, \underset{\sim}{\zeta}, \Delta k)$ satisfies the following partial differential equation

$$\frac{\partial \Gamma_D(z, \underset{\sim}{\zeta}, \Delta k)}{\partial z} + \frac{i\Delta k}{2k^2}\,\nabla_{\underset{\sim}{\zeta}}^2\,\Gamma_D + \frac{1}{2k^2}\,[A_\beta(0) - A_\beta(\underset{\sim}{\zeta})]\,\Gamma_D = 0 \tag{13}$$

The initial condition for Γ_D is $\Gamma_D\,(z = 0) = 1$. Γ_R refers to the "pure refraction" effect because it represents the result of different transit times of different ray paths due to a varying index of refraction. Γ_D refers to the "diffraction effect" arising from the diffraction term $\left(\frac{\nabla_1^2}{k_1} - \frac{\nabla_2^2}{k_2}\right)\Gamma_{1,1}$ in Eq. (10), or $\nabla_{\underset{\sim}{\zeta}}^2\,\Gamma_D$ in Eq. (13).

In order to proceed further, we have to specify the two-point correlation of the fluctuating index $\varepsilon_k(\underset{\sim}{r})$ (or $\beta(\underset{\sim}{r})$), from which the function $A_\beta(\underset{\sim}{\zeta})$ appearing in Eq. (10) or Eq. (13) can be obtained through Eq. (11). Since the medium is statistically homogeneous, the correlation function $<\varepsilon_k(\underset{\sim}{r})\ \varepsilon_k(\underset{\sim}{r}')>$ depends only on $|\underset{\sim}{r} - r'|$. It is convenient here to work with the spatial power spectrum $P_\varepsilon(\underset{\sim}{q})$ of the fluctuations, which is related to the correlation through

$$P_\varepsilon(\underset{\sim}{q}) = \int d^3\underset{\sim}{r} < \varepsilon_k(\underset{\sim}{x})\ \varepsilon_k(\underset{\sim}{x} + \underset{\sim}{r})> e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}} . \tag{14}$$

We consider two types of power spectrum for $P_\varepsilon(\underset{\sim}{q})$: a Gaussian spectrum and a power-law spectrum. For a Gaussian spectrum, $P_\varepsilon(q)$ is written as

$$P_\varepsilon(q) = B_G\ e^{-q^2 L_G^2} \qquad \text{(Gaussian)} \tag{15a}$$

where L_G is the correlation scale of the fluccuations in refractive index. For a power-law spectrum, $P_\varepsilon(q)$ has the following form,

$$P_\varepsilon(q) = B_p \frac{e^{-q^2 \ell_p^2}}{(1 + q^2 L_p^2)^{\alpha/2}} \qquad \text{(Power-law)} \tag{15b}$$

where L_p is the correlation scale of index fluctuations and ℓ_p is termed the cut-off (or inner) scale. Usually $\ell_p << L_p$, and equation (15b) for $\alpha = \frac{11}{3}$ is the Kolmogorov turbulence spectrum. Note that in each case B may be related to the local mean-square electron density fluctuations. We have

$$B_G = 128\ \pi^{7/2}\ r_e^2\ k^{-4}\ L_G^3 <\delta\ N_e^2> \tag{16a}$$

$$B_p = 128\ \pi^{7/2}\ r_e^2\ k^{-4}\ L_p^3 <\delta N_e^2>\ \Gamma(\tfrac{\alpha}{2})/\Gamma(\tfrac{\alpha}{2} - \tfrac{3}{2}) \tag{16b}$$

where $r_e = \frac{e^2}{mc^2}$ is the classical electron radius and $<\delta N_e^2>$ is the r.m.s fluctuating electron density.

For the Gaussian spectrum of Eq. (15a), we have from Eqs. (11), (14)

$$A_\beta(\zeta) = \frac{B_G k^4}{8\pi L_G^2} e^{-\left(\frac{\zeta^2}{4L_G^2}\right)}$$

$$\simeq \frac{B_G k^4}{8\pi L_G^2}\left(1 - \frac{\zeta^2}{4L_G^2}\right) \quad (\zeta \ll L_G). \tag{17}$$

Next we consider the power-law spectrum of Eq. (15b). For $|\zeta| \gg \ell_p$, we neglect the effect of the cutoff at $q > \ell_p^{-1}$ and obtain from Eqs. (11), (14) and (15b)

$$A_\beta(\zeta) = \frac{B_p k^4}{L_p^2\, 2^{2+\mu}\, \pi} \frac{(\zeta/L_p)^\mu\, K_\mu(\zeta/L_p)}{\Gamma(\mu+1)} \tag{18}$$

where $\mu = \frac{\alpha}{2} - 1$, $2\mu + \frac{3}{2} > 0$, and K_μ denotes a modified Bessel function of the second kind. One can further show that for $L_p \gg \zeta$,

$$A_\beta(\zeta) = \frac{\beta_p k^4}{4\pi(\alpha-2)L_p^2}\left[1 - \frac{\Gamma(2-\frac{\alpha}{2})}{\Gamma(\frac{\alpha}{2})}\left(\frac{\zeta}{2L_p}\right)^{\alpha-2}\right] \tag{18a}$$

From Eq. (17) or (18a), we may write

$$D_\beta(\zeta) = A_\beta(0) - A_\beta(\zeta)$$

$$\simeq \beta_o\, \zeta^\nu, \tag{19}$$

and thus Eq. (13) becomes

$$\frac{\partial\Gamma_D}{\partial z} + \frac{i\Delta k}{2k^2}\nabla_\zeta^2\,\Gamma_D + \frac{\beta_o\,\zeta^\nu}{2k^2}\Gamma_D = 0 \tag{20}$$

For the Gaussian spectrum, $\nu = 2$ and

$$\beta_o = \frac{B_G\, k^4}{32\pi L_G^4}. \tag{21a}$$

For power-law spectrum with $2 < \alpha < 4$, $\nu = \alpha - 2$ and

$$\beta_o = \frac{B_p\, k^4\, \Gamma(2-\frac{\alpha}{2})}{4\pi(\alpha-2)\,2^{(\alpha-2)}\,\Gamma(\frac{\alpha}{2})\, L_p^\alpha}. \tag{21b}$$

Eq. (20) can be solved numerically (see Appendix, Paper II). The numerical results for Γ_D, together with the values of Γ_R in Eq. (12a), give us the values of $\Gamma_{1,1}$, which are then inserted into Eqs. (8') and (9') to obtain the cross-correlation function $P_I(z, \underline{\zeta}_1, k_1, \underline{\zeta}_2, k_2)$ and the intensity correlation of finite bandwidth, Q_I $(z, \underline{\zeta}_1, \underline{\zeta}_2, K)$.

Before presenting the numerical results, let us consider the "pure refraction" effect, Γ_R $(z,\Delta\kappa)$. This effect can be neglected if

$$\left|\frac{\Delta k^2}{2k^2} A_\beta(o) z\right| << 1 \ . \tag{22}$$

For a Gaussian spectrum, the condition in Eq. (22) is easily satisfied in interstellar scintillations and the pure refraction effect can be neglected. For a Kolmogorov spectrum ($\alpha = \frac{11}{3}$) in interstellar scintillations, Eq. (22) is not satisfied and the "pure refraction" effect Γ_R is important. However, as discussed in Paper II, the "pure refraction" effect is caused by variation of the optical path along the line of sight and will <u>not</u> be observed at a position essentially stationary with respect to the scattering medium. In observations carried out over a period of hours or days, therefore, an Earth-bound observer could hope to detect only the diffraction effect Γ_D. Thus we will neglect the "pure refraction" effect in the following presentation. In any case, this effect could easily be incorporated into the calculation of P_I or Q_I because of its simple analytic form in Eq. (12a).

The numerical results for P_I $(z, \underline{\zeta}_1, k_1, \underline{\zeta}_2, k_2)$ are plotted in Figures (1), (2) and (3). Figure (1) shows the frequency correlation of intensity fluctuations,

$$P_I(\Delta f) = P_I\left(z, \underline{\zeta}_1, k_1, \underline{\zeta}_2 = \underline{\zeta}_1, k_2 = k_1 - \frac{2\pi\Delta f}{c}\right) \tag{23}$$

as a function of the normalized frequency separation $F = (\Delta f/f_D)$ between two detectors with frequency f_1 and $f_2 (=f_1 - \Delta f)$ respectively. Curve (1) in figure (1) is for a turbulent medium with a Kolmogorov spectrum and curve (2) for a Gaussian spectrum. f_D is the "decorrelation frequency," which is defined as the frequency difference of two observing channels beyond which the intensities measured at these two channels are essentially uncorrelated. For an observer at z, we have the decorrelation frequency

$$f_D = 2\pi c\ \beta_o^{-2/\nu}\ k^{2(\nu + 2)/\nu} \left(\frac{z}{2}\right)^{-(\nu + 2)/\nu} \tag{24}$$

For a Gaussian spectrum $\nu = 2$, $f_D \propto \lambda^{-4}$ and β_o is given by Eq. (21a), while for a Kolmogorov spectrum, $\nu = \frac{5}{3}$, $f_D \propto \lambda^{-4.4}$ and β_o is given by Eq. (21b). Note that the decorrelation frequency f_D is related to the characteristic time t_D for pulse broadening (Paper II) and the characteristic scattering angle Θ_c (Paper I) respectively by

$$2\pi\ f_D\ t_D = 1 \tag{25a}$$

and

$$f_D = \frac{c}{\pi z \Theta_c^2} \tag{25b}$$

for both the Gaussian and Kolmogorov spectra.

Figures (2) and (3) show, respectively for the Gaussian and Kolmogorov spectra, the cross-correlation function $P_I(\zeta, F) = P_I\ (z, \underset{\sim}{\zeta}_1, k_1, \underset{\sim}{\zeta}_2\ k_2)$ as a function of $\zeta = |\underset{\sim}{\zeta}_1 - \underset{\sim}{\zeta}_2|$ for various values of the normalized frequency separation $F = \Delta f/f_D$. The characteristic intensity correlation scale ζ_c shown in the figures is related to the decorrelation frequency f_D by

$$\zeta_c = \frac{1}{k\ \Theta_c} \tag{26}$$

and Eq. (25b). ζ_c can then be written in terms of the characteristic parameters in the interstellar medium by Eq. (24). We only note that $\zeta_c \propto \lambda^{-1}\ z^{-0.5}$

$L_G^{0.5} \langle\delta N_e^2\rangle^{-0.5}$ for the Gaussian spectrum and $\zeta_c \propto \lambda^{-1.2} z^{-0.6} L_p^{0.4}$ $\langle\delta N_e^2\rangle^{-0.6}$ for a Kolmogorov spectrum. As shown in Figures (2) and (3), the spatial cross-correlation P_I becomes broader as the frequency separation F is increased. The broadening of P_I is caused by the preferential removal of the high-frequency components of its Fourier spectrum, a result of the incoherence between waves of different frequencies generated by the term $\Delta k \nabla_{\underset{\sim}{\xi}}^2 \Gamma_D$ in Eq. (13).

IV. BANDWIDTH EFFECTS

In order to calculate the intensity-correlation function Q_I for a detector with finite bandwidth, it is necessary to specify the bandwidth function $f_K(k)$. As an example, we consider a rectangular bandpass of frequency width Δf_o (or $\Delta k_o = 2\pi\Delta f_o/c$): $f_K(k) = \frac{1}{\Delta k_o}$ for $k_o - \frac{\Delta k_o}{2} \leq k \leq k_o + \frac{\Delta k_o}{2}$, and zero otherwise. Let $K \equiv \Delta f_o/f_D$ be the normalized bandwidth. The correlation function Q_I can be obtained by inserting the numerical results of $\Gamma_{1,1}$ into Eq. (8') and integrating over wavenumbers. Figures (4) - (6) show the results. The scintillation index m_Z as a function of the parameter K is plotted in Figure (4). Curve (1) is for a Kolmogorov spectrum and curve (2) for Gaussian spectrum. The scintillation index m_Z is the unity for small bandwidths and falls to a value of 0.5 for K = 5.

Figures (5) and (6) show the renormalized intensity correlation function $Q_N(\zeta,K) = Q_I(\zeta, K)/Q_I(\zeta,K = 0)$ as a function of $\zeta = |\zeta_1 - \zeta_2|$ and the bandwidth K for the Gaussian and the Kolmogorov spectra respectively. Note that the intensity correlation scale increases as the bandwidth K increases. (See also Little 1968). As in the case of the broadening of P_I, this effect is due to the preferential removal of high-frequency components arising from diffraction-generated incoherence.

Let us define ζ_h (K) be the half-width of the function Q_N (ζ, K). Figure (7) shows the percentage increase of ζ_h (K) relative to ζ_h (0) as a function of the bandwidth K. As shown in the Figure, the percentage increase of ζ_h (K) for a Kolmogorov spectrum is almost twice as large as that for a Gaussian spectrum. At K = 10, the increase of ζ_h for a Kolmogorov

spectrum is $\sim$ 60% while for a Gaussian spectrum, the increase is $\sim$ 30%. The interpretation for this difference can be found from the power spectra of index fluctuations shown in Figure (8). Curves (1) and (2) show respectively the Kolmogorov and the Gaussian spectra. The observed wave intensity fluctuations are caused mostly by the power inside the rectangle abcd in Figure (8), where the spatial frequency q is close to $(kz)^{-1/2}$, the inverse of the Fresnel scale. The components of higher spatial frequency in the intensity power spectrum, which are removed preferently because of the incoherence between different frequencies for a detector with finite bandwidth, arise largely from that portion of the power spectrum contained in the area cefg in Figure (8). Inside the shaded area, there is more power to be removed from a Kolmogorov spectrum than from a Gaussian spectrum. Hence the higher frequency power in intensity fluctuations is reduced more severely and the intensity correlation function Q_I becomes broader for a Kolmogorov spectrum than for a Gaussian spectrum. Thus the difference between a power-law spectrum and a Gaussian spectrum are amplified here by the finite bandwidth of a detector. The broadening of the half-width of Q_I can be used to determine whether the electron density fluctuations in the interstellar medium have a Gaussian or a power-law spectrum.

Finally we remark that an analysis of the increase of the intensity correlation scale has not yet been carried out for interstellar scintillations. The published data on interstellar scintillations are not sufficiently accurate to determine whether the

interstellar turbulent medium has a Gaussian or a Kolmogorov spectrum (Lee and Jokipii 1975 d), because most observed quantities are not very sensitive to the for of the power spectrum. However, the increase of the correlation scale due to finite bandwidth of a detector is relatively sensitive to the form of the interstellar power spectrum. Therefore, we suggest here measurement of the increase of intensity-correlation scale as a method to determine whether the power spectrum in the interstellar medium is Gaussian or Kolmogorov.

FIGURE CAPTIONS

Figure (1) This figure shows the frequency correlation of intensity fluctuations P_I (F) as a function of the normalized frequency separation $F = \Delta f/f_D$. f_D is the decorrelation frequency. Curve (1) is the for Kolmogorov spectrum and curve (2) for the Gaussian spectrum.

Figure (2) The cross-correlation function P_I (ζ,F) as a function of $\zeta = |\underset{\sim}{\zeta}_1 - \underset{\sim}{\zeta}_2|$ for various values of the normalized frequency separation $F=\Delta f/f_D$. ζ_c is the characteristic correlation scale. This figure is for a Gaussian turbulent spectrum.

Figure (2) As in Figure (2) for the medium with a Kolmogorov spectrum.

Figure (4) The scintillation index m_z (K) is plotted as a function of the parameter K (normalized bandwidth). Curves (1) and (2) are respectively for a Kolmogorov and a Gaussian spectrum.

Figure (5) This figure shows the renormalized intensity correlation function Q_N (ζ,K) as a function of $\zeta = |\underset{\sim}{\zeta}_1 - \underset{\sim}{\zeta}_2|$ for various values of the normalized bandwidth K for a medium with a Gaussian spectrum.

Figure (6) As in Figure (5) for the medium with a Kolmogorov spectrum.

Figure (7) This figure shows the percentage increases of the half-width ζ_h (K) of intensity correlation relative to ζ_h (K = 0) as a function of the bandwidth K for both Gaussian and Kolmogorov spectra.

Figure (8) This figure shows the power spectrum of the refractive index, P_ε (q). Curve (1) is a Kolmogorov spectrum and curve (2) is a Gaussian spectrum. The intensity fluctuations of radio waves are caused mostly by the power inside the rectangle abcd. The components of highest spatial frequency in the intensity power spectrum are mostly due to the power in the shaded area cefg. This figure is plotted in a log-log scale.

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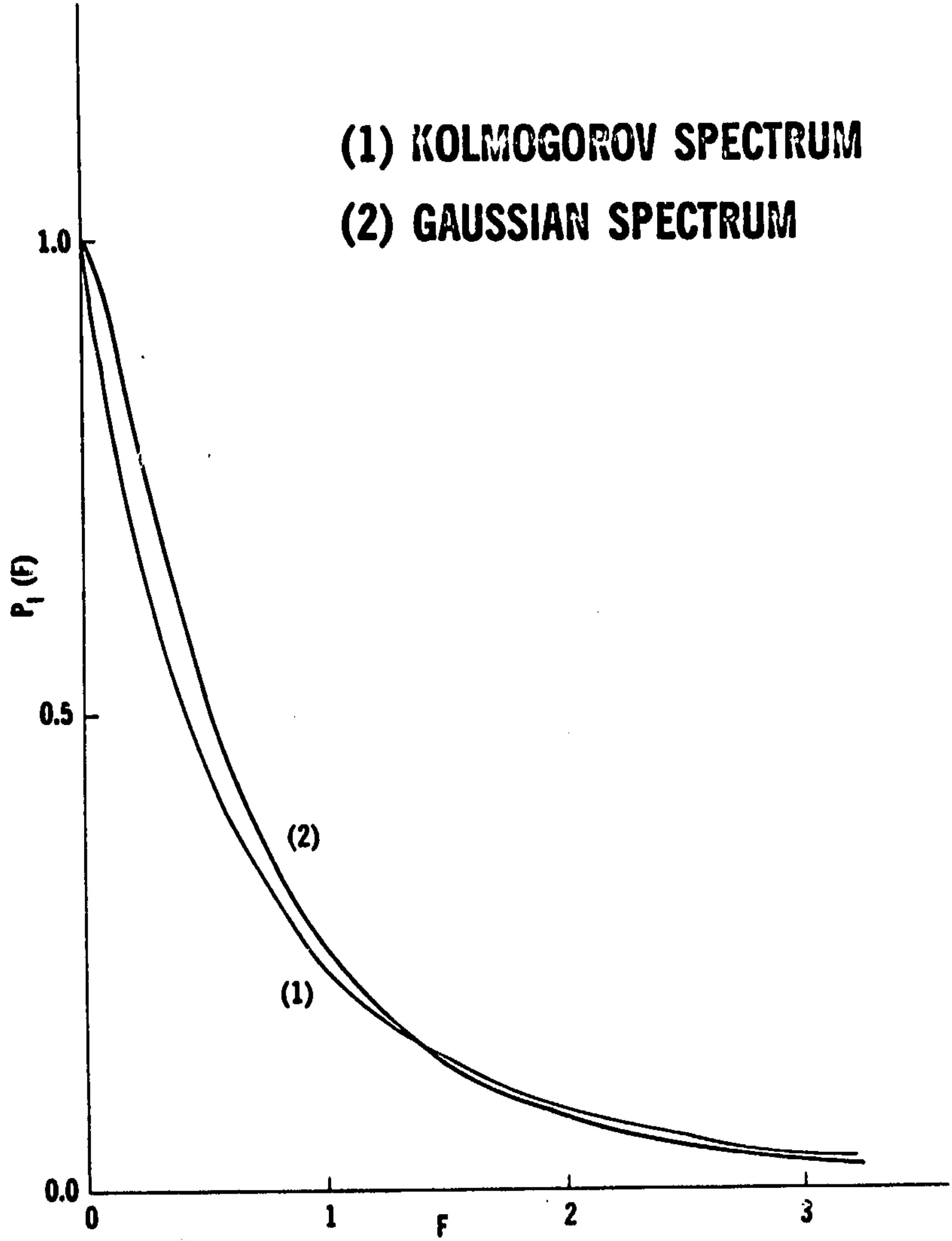

Figure (1)

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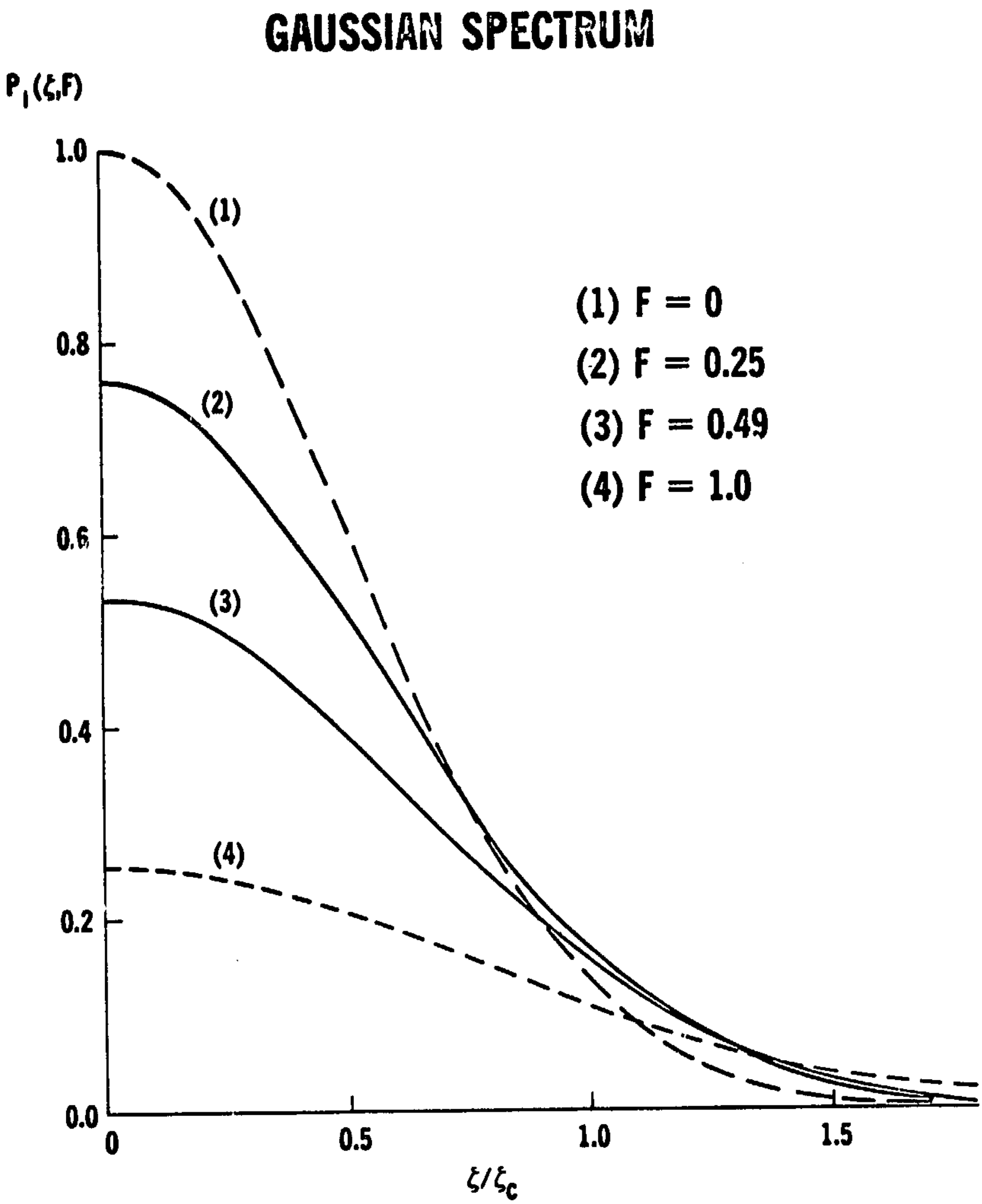

GAUSSIAN SPECTRUM
P1(ζ,F)
1.0
0.8
0.6
0.4
0.2
0.0
(1)
(2)
(3)
(4)
(1) F = 0
(2) F = 0.25
(3) F = 0.49
(4) F = 1.0
0
0.5
1.0
1.5
ζ/ζc

Figure (2)

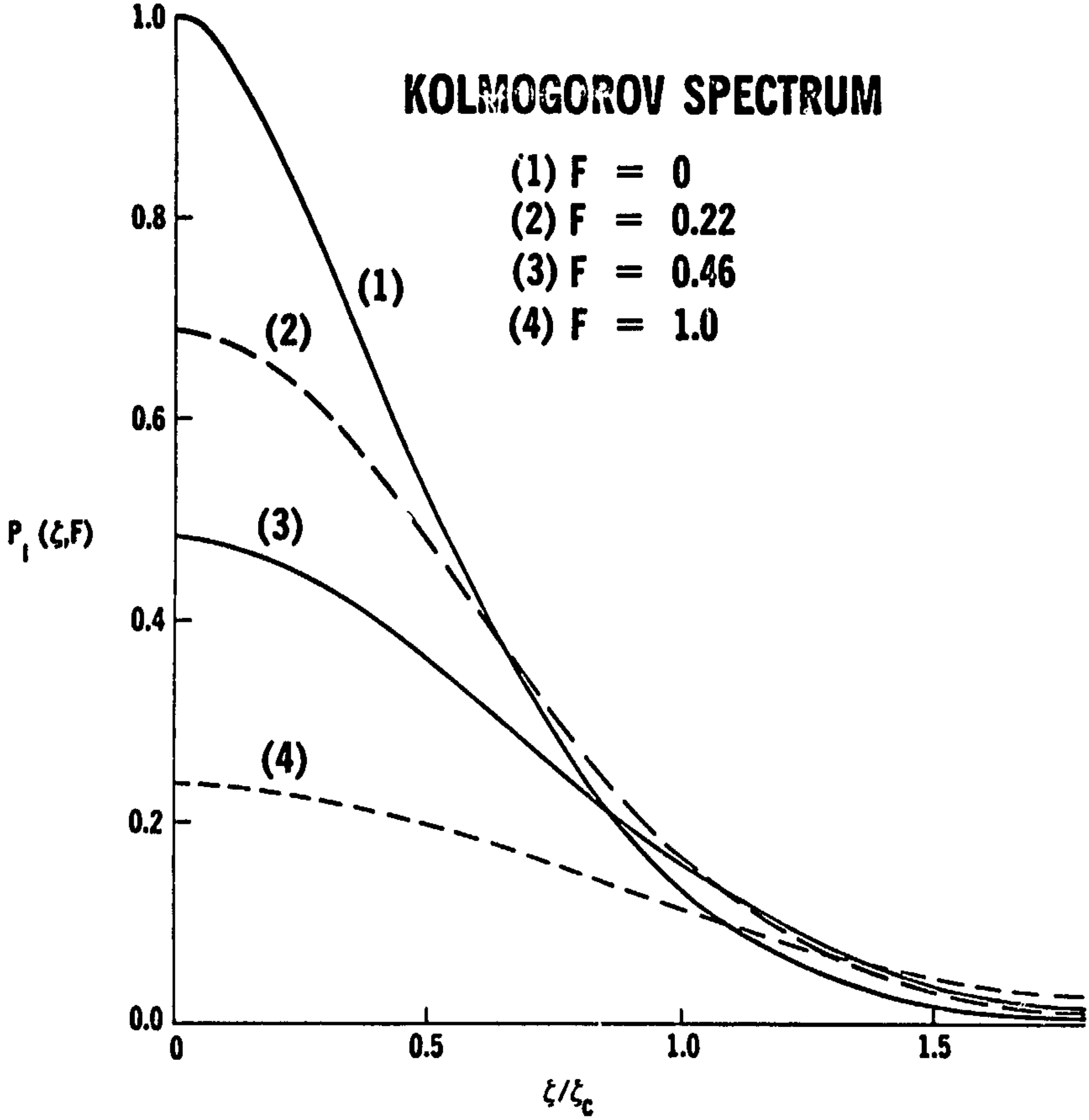
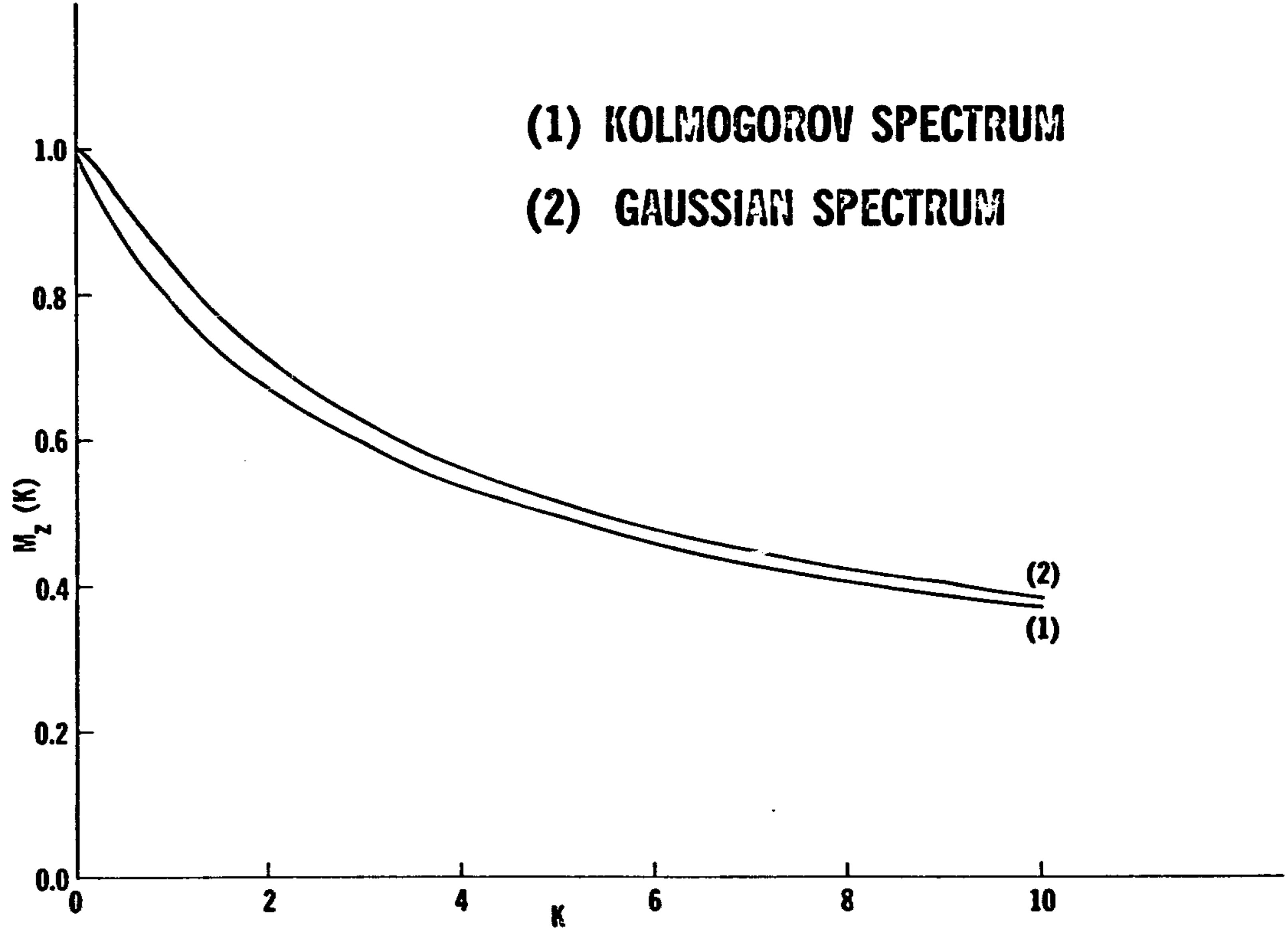
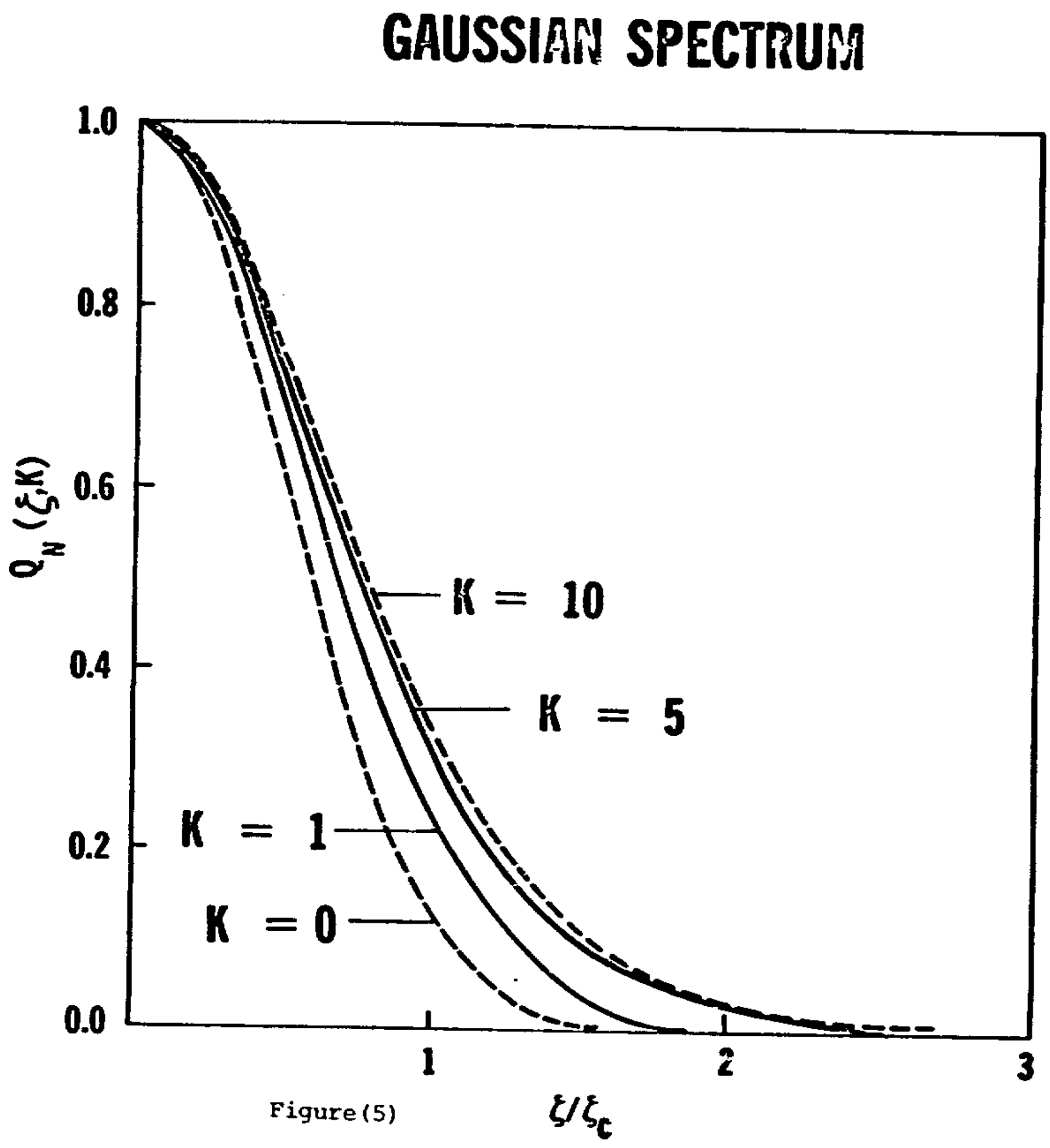

1.0
0.8
0.6
0.4
0.2
0.0
0
0.5
1.0
1.5
KOLMOGOROV SPECTRUM
(1) F = 0
(2) F = 0.22
(3) F = 0.46
(4) F = 1.0
(1)
(2)
(3)
(4)
P1 (ζ,F)
ζ/ζc

Figure (3)



(1) KOLMOGOROV SPECTRUM
(2) GAUSSIAN SPECTRUM
1.0
0.8
0.6
0.4
0.2
0.0
M_z (K)
0
2
4
6
8
10
K
(2)
(1)

Figure (4)



GAUSSIAN SPECTRUM
$Q_N(\zeta,K)$
1.0
0.8
0.6
0.4
0.2
0.0
K = 10
K = 5
K = 1
K = 0
1
2
3
ζ/ζ_c

Figure(5)

KOLMOGOROV SPECTRUM

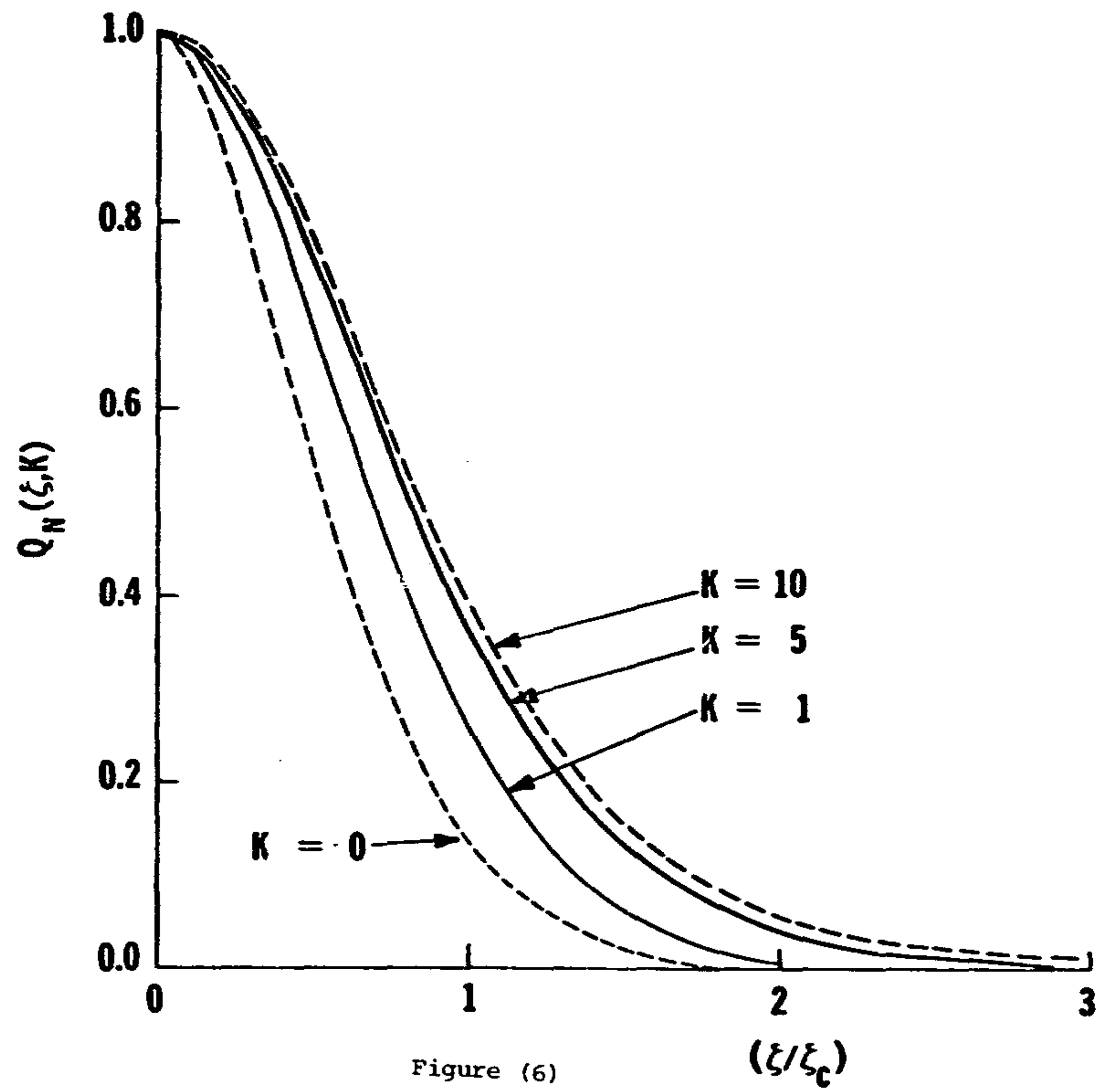


Figure (6)

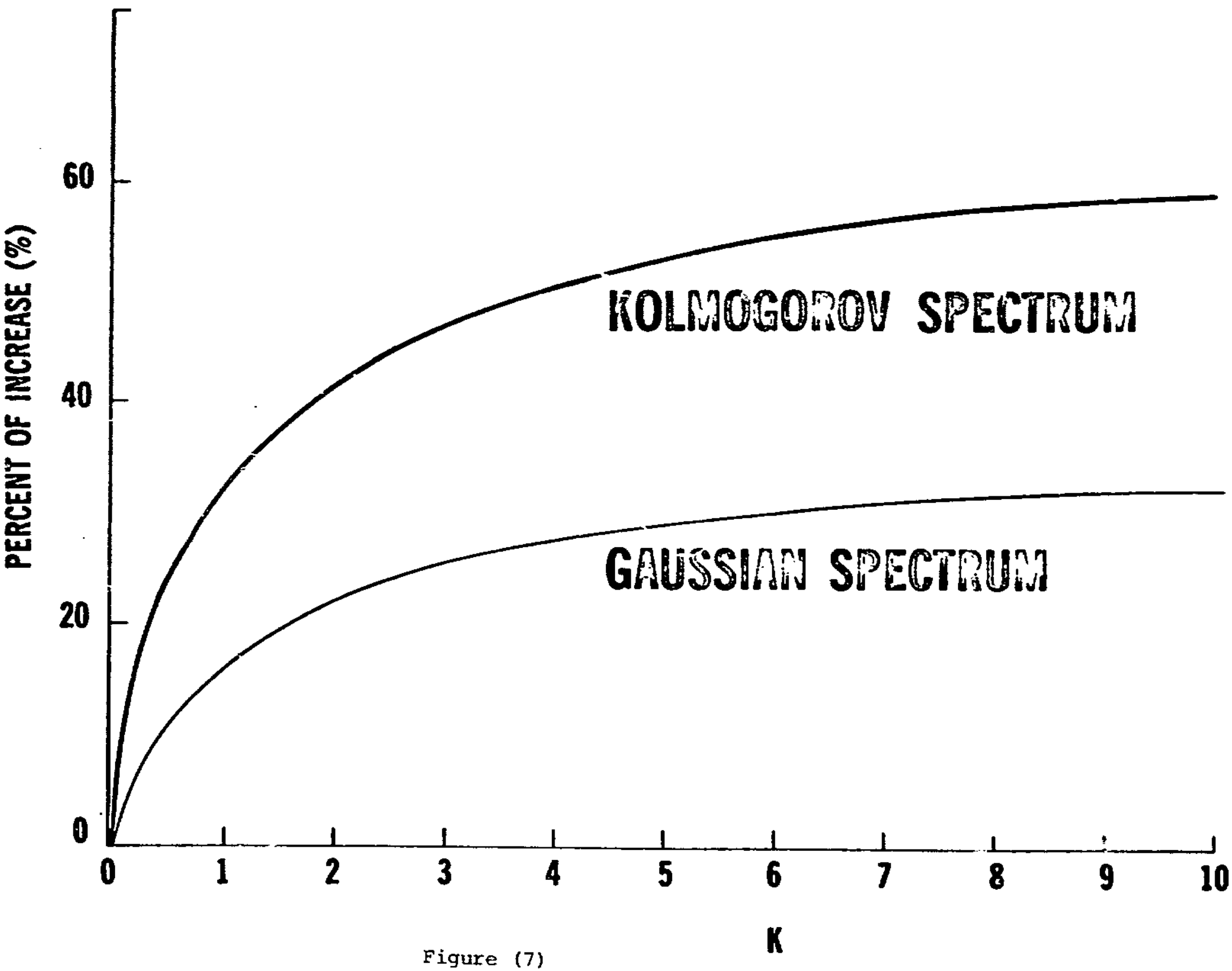

PERCENT OF INCREASE (%)
60
40
20
0
KOLMOGOROV SPECTRUM
GAUSSIAN SPECTRUM
0
1
2
3
4
5
6
7
8
9
10
K

Figure (7)

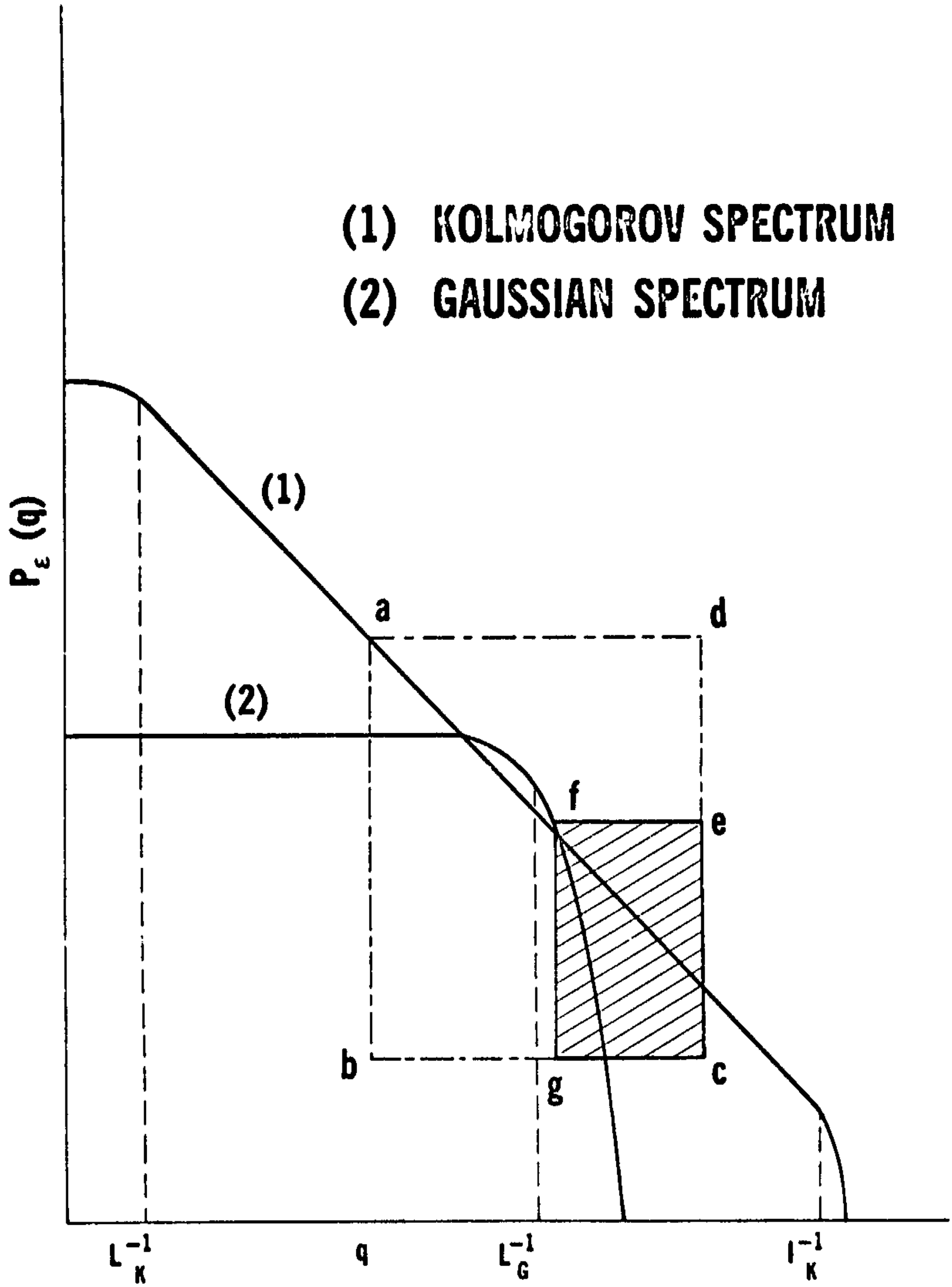

(1) KOLMOGOROV SPECTRUM
(2) GAUSSIAN SPECTRUM
(1)
(2)
P_ε (q)
a
d
f
e
b
g
c
L_K^{-1}
q
L_G^{-1}
l_K^{-1}

Figure (8)